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need be rejected to get a Riemannian geometry, the "Single Elliptic Geometry."

Boyer has perhaps been misled by his own paraphrases for what really occurs in Euclid. The real postulatum is as follows: "And if a straight cutting two straights makes with them angles interior and lieing on the same side, which together are less than two right angles, then the two straights indefinitely produced cut each other on the side on which these angles lie."

Needless to say, this remains true in both single and double elliptic geometry. The postulate currently taken in place of the unwieldy postulatum of Euclid is "that two straight lines, which cut one another, cannot be both parallel to the same straight line," which is credited by Playfair to Ludlam, though attributed even by Cajori to Playfair, and currently called Playfair's axiom. Even this does not help toward Riemannian spaces, for in them there are no parallels in the sense of coplanar non-meeting straights.

Finally Boyer gives us the well-known confusion in connection with Beltrami's pseudo-sphere.

John Bolyai found a surface in Lobachevskian space whose geodesic geometry is that of Euclid's straights. No one supposed that this reduced Euclidean to be only a branch of Bolyaian geometry.

Beltrami found a surface in Euclidean space whose geodesic geometry is that of Bolyai's straights. Boyer says, page 246, this reduced the Geometry of Lobachevski to be only a branch of ordinary geometry. On the contrary, the truth is that Euclidean geometry is only that special case of Bolyaian geometry made by assuming the space-constant as infinite.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

134. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

A certain piece of land is surrounded by a four-board fence, the boards being 16 feet long. The number of acres in the land equals the number of boards in the fence. How many acres in the land?

I. Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; P. S. BERG, B. Sc., Larimore, N. D.; and MARTIN SPINKS, Wilmington, O.

(1) *When the field is in the form of a square.*

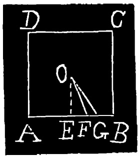
Let $ABCD$ be the square, O its center, $FG=l=16$ feet=the length of a panel of the fence, and $n=4$=the number of boards in a panel. Then the area of the triangle $OFG=\frac{1}{2}OE\times FG=\frac{1}{2}OE.l=n$ acres.

$\therefore OE=\dfrac{2\times43560\times n}{l}$, and AB, a side of the field$=2OE=\dfrac{4\times43560\times n}{l}$.

Hence the area of the field$=\left(\dfrac{4\times43560\times n}{l}\right)^2\div43560=\dfrac{16\times43560\times n^2}{l^2}$.

$=43560$ acres.

SPINKS.



$(\text{side})^2/160=$number of acres in tract.

$4\times16\frac{1}{2}\times\text{side}=$number of feet in perimeter of field.

$4\left(\dfrac{4\times16\frac{1}{2}\times\text{side}}{16}\right)=$number of boards in fence.

$\therefore \dfrac{(\text{side})^2}{160}=\dfrac{4(4\times16\frac{1}{2}\times\text{side})}{16}=16\frac{1}{2}\times\text{side}$.

$\therefore$ side$=2640$ rods$=8\frac{1}{4}$ miles.

$(2640)^2\div160=43560$ acres.

ZERR AND BERG.

(2) *When the field is in the form of a circle.*

Let $R=$radius in rods.

$\therefore \pi R^2/160=$area in acres.

$2\pi R\times16\frac{1}{2}=$perimeter in feet.

$4\left(\dfrac{2\pi R\times16\frac{1}{2}}{16}\right)=$number of boards in perimeter.

$\therefore \dfrac{\pi R^2}{160}=\dfrac{4(2\pi R\times16\frac{1}{2})}{16}=8\frac{1}{4}\pi R$.

$\therefore R=1320$ rods.

$\pi R^2/160=10890\pi$ acres.

II. **Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Let $16m(p^2+q^2)x$ and $16nx=$the respective sides, in feet, of a *parallelogram*. Then the number of boards in the fence

$$=4\times\tfrac{1}{16}\times32[m(p^2+q^2)+n]x=8[m(p^2+q^2)+n]x\ldots.(1).$$

For *rectangles*, the number of acres

$$=\frac{(16nx)[16m(p^2+q^2)x]}{43560}=\frac{32mn(p^2+q^2)x^2}{5445},$$

which equals (1).

$$\therefore x=\frac{5445[m(p^2+q^2)+n]}{4mn(p^2+q^2)}\ldots.(2).$$

For *rhomboids*, the altitude$=16m(p^2-q^2)x$ or $32mpqx$; and number of acres

$$=\frac{32mn(p^2-q^2)x^2}{5445}\text{ or }\frac{64mnpqx^2}{5445}.$$

Equating these two values with (1) and reducing, we find

$$x=\frac{5445[m(p^2+q^2)+n]}{4mn(p^2-q^2)}\ldots.(3),\text{ and } x=\frac{5445[m(p^2+q^2)+n]}{8mnpq}\ldots.(4).$$

p, q, and m may be any integers. $p>q$. Also, p and q may be such fractions that, in connection with m and n, will render x integral or of such values as to have the sides divisible by 16.

(a) Take $p=\frac{4}{5}$, $q=\frac{3}{5}$, $m=1$; and substitute in (2), (3), and (4).

Then $x=\frac{5445(1+n)}{4n}\ldots.(5)$, $x=\frac{136125(1+n)}{28n}\ldots.(6)$,

$x=\frac{45375(1+n)}{16n}\ldots.(7)$, and (1) becomes $8(1+n)x$.

x must be integral.

From the factors of 5445, we observe that, in (5), $n=3$, 11, 15, 55, 99, 363, $\frac{1}{3}$, $\frac{5}{3}$, $\frac{3}{5}$, $\frac{11}{5}$, $\frac{5}{11}$, $\frac{9}{11}$, etc.; in (6), $n=55$, 363, $\frac{1}{55}$. etc.; and in (7), $n=15$, $\frac{1}{15}$, etc.

Tne following table of values explains itself.

n	*Eq.*	x	*Boards=Acres*	*Sides in Feet.*
3	(5)	1815	58080	29040 and 87120
11	(5)	1485	142560	23760 and 261360
$\frac{9}{11}$	(5)	3025	44000	48400 and 39600
15	(5)	1452	185856	23232 and 348480
15	(7)	3025	387200	48400 and 726000
55	(5)	1357	620928	22176 and 1219680
55	(6)	4950	2217600	79200 and 4356000
etc.	etc.	etc.	etc.	etc.

(b) Take $p=2$, $q=m=1$; and substitute in (2), (3), and (4).

Then $x=\frac{1089(5+n)}{4n}\ldots.(8)$, $x=\frac{1815(5+n)}{4n}\ldots(9)$, $x=\frac{5445(5+n)}{16n}\ldots(10)$, and (1) becomes $8(5+n)x$.

Whence the following table :

n	*Eq.*	x	*Boards=Acres*	*Sides in Feet*
3	(8)	726	46464	58080 and 34848
3	(9)	1210	77440	96800 and 58080
11	(8)	396	50688	31680 and 69696
11	(9)	660	84480	52800 and 116160
11	(10)	605	77440	48400 and 106480
$\frac{9}{11}$	(8)	1331	61952	106480 and 17424
$\frac{9}{11}$	(10)	2420	112640	193600 and 31680
etc.	etc.	etc.	etc.	etc.

(c) And so on for all other values of p, q, and m.

Mr. Gruber sent in three different solutions, and Mr. D. B. Northrup, of Mandana, N. Y., sent in the results for tracts in the form of (1) circle, (2) square, (3) rectangle sides as 2:1, (4) a triangle ratios of side 6:6:7, and (5) an ellipse having a major-axis double the minor-axis. Other very simple solutions of the problem are possible when the tract is in the shape of a square, but the solutions above are quite sufficient for all purposes. ED. F.

136. Proposed by F. M. PRIEST, Mona House, St. Louis, Mo.

What is the size of the smallest cubical box, inside dimension, that will contain four balls each ten inches in diameter?

Solution by H. N. DAVIS, (Brown University), 159 Brown Street, Providence, R. I.

Let the base of the box be of such a size that two of the balls when placed with their centers along a diagonal will be tangent to each other and to the sides of the box. Let their centers be C and C', the diagonal AD, and the center O.

Draw BC perpendicular to AE. Then $AB=BC=r=5$.

$\therefore AC=5\sqrt{2}$.

$AO=5\sqrt{2}+CO=5\sqrt{2}+r=5\sqrt{2}+5$.

$AD=10(1+\sqrt{2})=\sqrt{2}AE$.

$AE=5\sqrt{2}+10=17.07106$ inches (nearly).

If the second layer of two balls be placed along the other diagonal (at $M+M'$) the position of M with reference to C and the side AF considered as a base will be exactly the same as that of C and C' with reference to AD. The figure may then be taken as an elevation and the height of the box will be exactly equal to its base-edge, and the box will be a cube. Q. E. D.

Good solutions were received from *PROF. J. M. STRASBURG*, Chicago, Ill., *D. B. NORTHRUP*, *G. B. M. ZERR*, and *M. A. GRUBER*.

ALGEBRA.

112. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

In *Hall and Knight's Higher Algebra* I find the following :

If $a+b+c=0$, then

$$\frac{a^5+b^5+c^5}{5}=\frac{a^3+b^3+c^3}{3}\cdot\frac{a^2+b^2+c^2}{2};\quad \frac{a^7+b^7+c^7}{7}=\frac{a^5+b^5+c^5}{5}\cdot\frac{a^2+b^2+c^2}{2};$$

and if $a+b+c+d=0$, then

$$\frac{a^5+b^5+c^5+d^5}{5}=\frac{a^3+b^3+c^3+d^3}{3}\cdot\frac{a^2+b^2+c^2+d^2}{2}.$$

QUERY. Is there a general law governing such expressions? Investigate.

I. Solution by HARRY S. VANDIVER, Bala, Montgomery County, Pa.

Consider the following general equation with its second term missing, thus